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ON THE CONTROL OF A RIGID BODY ROTATIONAL MOTION WITH INCOMPLETE INFORMATION ON THE ANGULAR VELOCITY VECTOR

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The present paper proves the possibility of controlling the rotational motion of a rigid body when the information on the current phase state of the system is incomplete and the only measurable parameter at each instant of time is the projection of the angular velocity vector on the axis of response of a single spin rate sensor rigidly attached to the object. Two control problems are considered: (1) imparting to the rigid body the mode of steady rotation with respect to the axis possessing the largest moment of inertia, the mode used in the system of passive stabilization [1–3], and (2) deceleration of a rotating rigid body. Conditions of stability of the above two modes of motion are analyzed.

1. Formulation of the problem. Introducing the xyz-coordinate system attached to the body, we assume that the response axis of the spin rate sensor coincides with the direction λ, the orientation of which with respect to the xyz basis is defined by the direction cosines α, β, γ and is assumed known.

Neglecting the dynamic properties of the sensor, we have the following signal at its output:

$$\vartheta = p\cdot\omega; \quad p = \{\alpha, \ \beta, \ \gamma\}, \quad \omega = \{\omega_x, \ \omega_y, \ \omega_z\} \tag{1.1}$$

where p is the unit vector of the λ-direction and ω is the angular velocity vector of the object. We agree to describe the body rotational motion in terms of the dynamic Euler equations (symbols within the brackets indicate that the remaining two equations are obtained by cyclic permutation of the indices)

$$I_x\omega_x^{\boldsymbol\cdot} + (I_z - I_y)\ \omega_y\omega_z = M_x \quad (xyz) \tag{1.2}$$

in which I_x, I_y, I_z are the moments of inertia (we shall assume, for definiteness, that $I_x < I_y < I_z$) and M_x, M_y, M_z are the control moments).

Problem 1. Using the information available in the form of (1.1), to formulate the control $M = \{M_x, M_y, M_z\}$, ensuring the asymptotic stability of the steady rotation of the object about the axis possessing the largest moment of inertia

$$\omega_x = \omega_y = 0, \quad \omega_z = \Omega = \text{const} \tag{1.3}$$

Problem 2. Using the information (1.1), to synthesize a control moment M ensuring the stability of the unperturbed motion

$$\omega_x = \omega_y = \omega_z = 0 \tag{1.4}$$

2. On controlling the motion when the information on the phase state of the system is incomplete. Consider the control system

$$x^{\boldsymbol\cdot} = f(x, u) \equiv Ax + F(x) + Bu \tag{2.1}$$

$$y = Cx, \quad f(0,\ 0) = 0$$
$$x \in R_n, \quad u \in U \subset R_m, \quad y \in R_s$$

Here A, B, C are constant matrices and $F(x)$ is a vector function the expansion of which in the powers of x_i $(i = 1, 2, \ldots, n)$ begins with terms of at least second degree. With the information about the motion of the object available in the form $y = Cx$, it is required to choose a control u in the way which would ensure the asymptotic stability of the unperturbed motion $x = 0$.

We shall assume that for the first approximation system

$$x^{\cdot} = Ax + Bu, \quad y = Cx \tag{2.2}$$

the pair (A, B) is controlled, and the pair (A, C) is an observed one. This implies that the nonlinear system (2.1) can be observed near the unperturbed motion [4] and be stabilized with the following linear control [5]:

$$u = Kx \tag{2.3}$$

The control in the form (2.3) can be realized directly from the results of the measurements of the observable parameters only in the case when a matrix G exists such that

$$K = GC \tag{2.4}$$

If such a matrix does not exist, we can solve the problem using a system of estimate of the state which will enable us to estimate all the components of the vector x from the incomplete information available about the state of the system, and will make it possible to formulate the control not with respect to the vector x, but with respect to its estimated value z in accordance with the expression

$$u = Kz \tag{2.5}$$

Assuming that the rank of the observability matrix

$$W = \| C' \ A'C' \ldots (A')^{n-1} \ C' \|$$

is n, we introduce the system of estimating the state in the form

$$z^{\cdot} = Az + l(y - Cz) + F(z) + Bu \tag{2.6}$$

From (2.1) and (2.6) it follows that the vector of the error $e = x - z$ satisfies the equation

$$e^{\cdot} = (A - lC)\, e + \Psi(x, e), \quad \Psi(x, e) = F(x) - F(z) \tag{2.7}$$

We note that when the initial conditions of the filter coincide with those of the object, the system (2.6) ensures that the exact recovery of the state vector x.

Next we shall find what requirements must the matrices K and l satisfy in order that the position of equilibrium $x = 0$ be asymptotically stable.

Let us consider the object equation, the system of estimating its state, and the control law

$$\begin{aligned} x^{\cdot} &= Ax + F(x) + Bu, \quad y = Cx \\ z^{\cdot} &= Az + l(y - Cz) + F(z) + Bu, \quad u = Kz \end{aligned} \tag{2.8}$$

Passing in the system (2.8) from the variables x, z to the variables x, e, we write it in the form

$$\begin{Vmatrix} x^{\cdot} \\ e^{\cdot} \end{Vmatrix} = \begin{Vmatrix} A + K & -K \\ 0 & A - lC \end{Vmatrix} \begin{Vmatrix} x \\ e \end{Vmatrix} + \begin{Vmatrix} F(x) \\ \Psi(x, e) \end{Vmatrix} \tag{2.9}$$

Analyzing (2. 9),we find that the problem of stability of the trivial solution $x = e = 0$ is reduced to that of the study of the stability of the matrices $A + K$ and $A - lC$. If the eigennumbers of these matrices have negative real parts, the unperturbed motion $x = e = 0$ of the system (2. 9) is not only stable, but also asymptotically stable [6]. Since in this case the vector of the error $e \to 0$ as $t \to \infty$, the system (2. 6) used to estimate the state becomes asymptotic.

3. Initial condition of the system (2. 6) of estimating the state. At the initial instant of time nothing is known on the state of the system (2. 6), therefore the usual assumption made is that $z(t_0) = 0$ [7]. However, a transitional process within the system of estimating the state may lead the controlled motion of the object to an unacceptable quality. To avoid this, we shall proceed as follows.

Let the object be undergoing an uncontrolled motion at $t < t_0$. To estimate the vector x of its phase state we introduce the system of equations

$$z^{\cdot} = Az + l(y - Cz), \quad y = Cx, \quad t \in [t_1, t_0], \quad z(t_1) = 0 \tag{3.1}$$

The error vector $e = x - z$ is governed in the present case by the equation

$$e^{\cdot} = (A - lC)\, e + F(x), \quad e(t_1) = x(t_1) \tag{3.2}$$

The system (3. 1) of estimation of the state is not asymptotic; however, if the matrix $A - lC$ is stable, the system allows the determination of the vector x with a definite accuracy depending on the manner in which the vector function $F(x)$ varies. The value of the vector z obtained by the instant $t = t_0$ from the solution of (3. 1), is used as the initial condition of the system (2. 6) of estimating the state.

4. Imparting a steady rotation to the body. We introduce the notation

$$a_1 = (I_y - I_z)\, I_x^{-1}, \quad u_1 = M_x I_x^{-1} \qquad (1, 2, 3, xyz) \tag{4.1}$$

$$\omega_x = x_1, \quad \omega_y = x_2, \quad \omega_z = x_3 + \Omega$$

and write Eq. (1. 2) in the form

$$x^{\cdot} = Ax + F(x) + u \tag{4.2}$$

$$x = \{x_1, x_2, x_3\}, \quad u = \{u_1, u_2, u_3\}$$

$$A = \begin{Vmatrix} 0 & a_1\Omega & 0 \\ a_2\Omega & 0 & 0 \\ 0 & 0 & 0 \end{Vmatrix}, \quad F(x) = \begin{Vmatrix} a_1 & x_2 & x_3 \\ a_2 & x_1 & x_3 \\ a_3 & x_1 & x_2 \end{Vmatrix}$$

We shall assume that the scalar

$$y = Cx, \quad C = \|\alpha\beta\gamma\| \tag{4.3}$$

represents the output of the system (4. 2).

Use of the above notation reduces the solution of Problem 1 to that of ensuring the asymptotic stability of the unperturbed motion $x = 0$.

It is not possible to realize the control in the form (2. 3) by observing the scalar y. We therefore assume that the following condition of observability of the system (4. 2), (4. 3) holds:

$$\det W \neq 0, \quad W = \|C' \; A'C' \; (A')^2 C'\| \tag{4.4}$$

This condition can be reduced to the requirement

$$a_1 a_2 \Omega^3 \gamma (a_2 \beta^2 - a_1 \alpha^2) \neq 0 \qquad (4.5)$$

and we shall control the rotational motion of the rigid body using the algorithm (2.5) and determining the vector z from the equation

$$z^{\cdot} = Az + l(y - Cz) + F(z) + u \qquad (4.6)$$

We note that the vector functions $F(z)$ and $\Psi(x, e)$ have, in the present case, the following form:

$$F(z) = \begin{Vmatrix} a_1 & z_2 & z_3 \\ a_2 & z_1 & z_3 \\ a_3 & z_1 & z_2 \end{Vmatrix}, \quad \Psi(x, e) = \begin{Vmatrix} a_1 (e_2 x_3 + e_3 x_2 - e_2 e_3) \\ a_2 (e_1 x_3 + e_3 x_1 - e_1 e_3) \\ a_3 (e_1 x_2 + e_2 x_1 - e_1 e_2) \end{Vmatrix}$$

Selection of the elements of the matrix K (we assume for simplicity that $K = \operatorname{diag}\{k_1, k_2, k_3\}$) such that the natrix $A + K$ is stable, presents no difficulties.

To ensure that the matrix $A - lC$ has eigennumbers specified beforehand, the vector l in the filter equation (4.6) must be chosen from the equalities

$$l = T^{-1}L, \quad T = \|t_1\, t_2\, t_3\|'$$
$$t_1 = (A')^2 C' + \alpha_1 A'C' + \alpha_2 C'$$
$$t_2 = A'C' + \alpha_1 C', \quad t_3 = C'$$

We denote by α_i the coefficients of the characteristic polynomial of the matrix A

$$\chi_A(s) = s^3 + \alpha_1 s^2 + \alpha_2 s + \alpha_3$$

It is important to note that $\det T = -\det W$, i.e. the vector l can be computed only when the system (4.2) is observable. Let $\theta(s) = s^3 + \beta_1 s^2 + \beta_2 s + \beta_3$ be an arbitrary normed polynomial the roots of which all have the negative real parts. If we now construct a vector L with components

$$L_{4-i} = \beta_i - \alpha_i \quad (i = 1, 2, 3)$$

the roots of the characteristic polynomial of the matrix $A - lC$ will coincide with the roots of the polynomial $\theta(s)$ [7].

Let $A + K$ and $A - lC$ be stable matrices. Then the control

$$M_x = I_x k_1 z_1 \quad (1\ 2\ 3,\ xyz) \qquad (4.7)$$

in which z_i $(i = 1, 2, 3)$ are found from the solution of (4.6), ensures the asymptotic stability of the steady rotation of the object about the axis possessing the largest moment of inertia.

5. Decelerating of a rotating rigid body. The results obtained in the course of solving Problem 1 can be extended to the case of deceleration of a rotating rigid body.

In fact, by specifying the accuracy of controlling the deceleration of the body we define the admissible residual angular velocity $\omega_z = \Omega_*$. Then the control in the form (4.6), (4.7) which is formulated under the assumption that $\Omega = \Omega_*$ guarantees the stability of the unperturbed motion (1.4) when the body is set by the control into the

the mode of steady rotation about the z-axis. In solving Problem 2, the body is set into the mode of steady rotation, it is not important about which axis the steady rotation is performed.

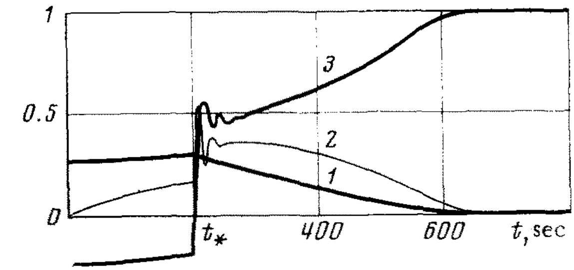

Fig. 1

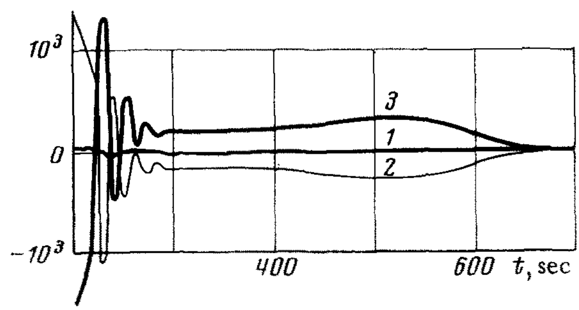

Fig. 2

6. Example. We consider the process of setting a rigid body with the following values of the inertia ellipsoid [8]:

$$I_x = 1.25 \cdot 10^6 \, \text{kg} \cdot \text{m}^2, \quad I_y = 6.9 \cdot 10^6 \, \text{kg} \cdot \text{m}^2, \quad I_z = 7.1 \cdot 10^6 \text{kg} \cdot \text{m}^2$$

from its initial state $\omega_x(0) = 0.25$ deg/sec, $\omega_y(0) = -0.25$ deg/sec, $\omega_z(0) = 0$ into the mode of steady rotation $\omega_x = \omega_y = 0$, $\omega_z = 1$ deg/sec. As the λ-direction along which the axis of response of the spin rate sensor is oriented, we take the direction which forms equal angles $(\alpha = \beta = \gamma = 1/\sqrt{3})$ with the x, y, z axes.

Figure 1 depicts the character of the variation in the angular velocities ω_x, ω_y, ω_z deg/sec (curves 1, 2 and 3, respectively). During the time interval $[0, t_*]$ the body performs an uncontrolled motion. The controlling moments (4.7) in which $k_1 = k_2 = k_3 = -1$ /sec are formed beginning from the instant $t = t_*$. The eigennumbers of the matrix $A - lC$ in the systems of estimating the state (3.1), (4.6) are assumed to be equal to $\mu_1 = \mu_2 = \mu_3 = -0.1$ /sec. The latter values have the corresponding vector l with components $l_1 = -3.62$ /sec, $l_2 = -37.8$ /sec. and $l_3 = 41.91$ /sec. Figure 2 depicts the variation in the controlling moments M_x, M_y, M_z n·m during the motion (curves 1, 2 and 3, respectively).

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